Finite-size effect in the Eguíluz and Zimmermann model of herd formation and information transmission

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The Eguíluz and Zimmermann model of information transmission and herd formation in a financial market is studied analytically. Starting from a formal description on the rate of change of the system from one partition of agents in the system to another, a mean-field theory is systematically developed. The validity of the mean-field theory is carefully studied against fluctuations. When the number of agents N is sufficiently large and the probability of making a transaction $a \ll 1/N \ln N$, finite-size effect is found to be significant. In this case, the system has a large probability of becoming a single cluster containing all the agents. For small clusters of agents, the cluster size distribution still obeys a power law but with a much reduced magnitude. The exponent is found to be modified to the value of -3 by the fluctuation effects from the value of $-5/2$ in the mean-field theory.

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I. INTRODUCTION

The model of herd formation and information transmission in a market introduced by Eguíluz and Zimmermann [1] (henceforth referred to as the EZ model) has received much attention recently [2–4]. The model considers a population of N agents. An agent can be connected to any of the $N-1$ other agents. The connectivity has the following properties. If Agent A is connected to B, then B is connected to A. If Agent A is connected to B and Agent B is connected to C, then A is connected to C. The connected agents form one cluster. In the beginning, all agents are not connected and the connectivity is established dynamically as follows. At each time step, an Agent A is selected at random. With a probability a, the connections in the cluster containing A are broken and all agents in this cluster become isolated agents. In the EZ model, a cluster of agents carry out the same action, i.e., buy or sell, with probability a and the cluster dissolves after a transaction is made. With a probability $1-a$, another Agent B is selected at random. If Agents A and B belong to two different clusters, then all the agents in the two clusters are connected together to form a bigger cluster. If Agents A and B happen to belong to the same cluster, no action is carried out and the next time step begins. EZ studied the cluster size distribution in the long-time limit and found that the number of clusters n_s of size s follows a power-law $n_s \sim s^{-5/2}$, with an exponential cutoff [1,2]. A price return can be mapped out from the collective action of the cluster of agents, when the cluster decides to make a transaction. The price return distribution also follows a power law with a behavior similar to that observed in some real markets [1,5,6]. The EZ model is a dynamical version of an earlier model of Cont and Bouchard [7] for herd formation in markets in which clusters are formed probabilistically with cluster size also follows a power law of exponent $-5/2$.

D'Hulst and Rodgers [2] developed a mean-field analysis for the EZ model and found an analytic expression for the size distribution of clusters of agents. Their analytic expression is valid when N is sufficiently large and $a \gg 1/\sqrt{N}$. Under these conditions, the state of the system corresponding to the entire population forming a single cluster is not important, and the finite-size effect is insignificant.

In the present paper, we study the EZ model analytically. Starting from a general description of the model through the rate of change of the system from one partition of agents into another, a mean-field theory can be developed systematically. The validity of the mean-field theory can be checked by considering the effects of fluctuations. Finite-size effect turns out to be significant, especially in the limit $a \ll 1/(N \ln N)$. When $a \ll 1/(N \ln N)$, in the limit $N \rightarrow \infty$, one would expect the state with a single cluster consisting of all the agents be dominant. The probability for other states to occur is small and proportional to a. However, it is of interest to find out the values of these probabilities in the case of $a \ll 1/(N \ln N)$, as the results will allow us to understand the finite-size effect in the intermediate regime of $a \sim 1/(N \ln N)$ in which the probability of having the state of a single cluster consisting of all the agents is finite but less than unity.

The paper is organized as follows. In Sec. II, we present the exact equations for describing the dynamics of the system. In the limit of $N \rightarrow \infty$ and $a \gg 1/\sqrt{N}$, we systematically develop a mean-field approach and recover the equations given by D'Hulst and Rodgers [2]. In Sec. III, we discuss the finite-size effect in the limit $N \rightarrow \infty$ and $a \ll 1/(N \ln N)$ within the mean-field approximation. In Sec. IV, we present the exact solutions to the EZ model in the limit of large N and $a \ll 1/(N \ln N)$. The validity of the mean-field solutions is checked against the exact solutions. The exact solutions are found to be slightly different from the mean-field solutions, indicating that fluctuation effect is significant. In Sec. V, we present the numerical results for different values of a. Results are summarized and discussed in Sec. VI.

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II. EXACT EQUATIONS

The dynamics of the EZ model can be described by considering the partition of N agents $[l_1,l_2,\ldots,l_N]$. Here, l_s is the number of clusters of size s $(s>0)$. It follows that

$$\sum_{i=1}^{N} il_i=N. \tag{1}$$

Since any state of a N-agent system can be characterized by a partition $[l_1,\ldots,l_N]$, the system can be described by the probability function $P[l_1,\ldots,l_N]$. The time evolution of $P[l_1,\ldots,l_N]$ is governed by the dynamics for cluster combination and dissociation as follows [8]:

$$\begin{aligned}\frac{dP[l_1,\ldots,l_N]}{dt}=&-\frac{1-a}{N(N-1)}\left[\sum_{i=1}^{N} il_i i(l_i-1)+\sum_{i<j} 2il_ijl_j\right]P[l_1,\ldots,l_N]+\frac{1-a}{N(N-1)}\left\{\sum_{i=1}^{N} i(l_i+2)i(l_i+1)\right.\\&\times P[l_1,\ldots,l_i+2,\ldots,l_{2i}-1,\ldots,l_N]+\sum_{i<j} 2i(l_i+1)j(l_j+1)P[l_1,\ldots,l_i+1,\ldots,l_j\\&\left.+1,\ldots,l_{i+j}-1,\ldots,l_N]\right\}-\frac{a}{N}\left\{\sum_{i=2}^{N} il_iP[l_1,\ldots,l_N]-i(l_i+1)\right.\\&\left.\times P[l_1-i,\ldots,l_i+1,\ldots,l_N]\right\}.\end{aligned} \tag{2}$$

The first four terms on the right-hand side of Eq. (2) describe the combination of clusters. The first term describes the reduction in $P[l_1,\ldots,l_N]$ due to the change from the partition $[,\ldots,l_i,\ldots,l_{2i},\ldots,]$ to the partition $[,\ldots,l_i-2,\ldots,l_{2i}+1,\ldots,]$ when two different clusters of the same size i are combined to form a larger cluster of size $2i$. The factor $il_ii(l_i-1)/N(N-1)$ is the probability of selecting two agents belonging to two different clusters of size i. Similarly, the second term describes the change from the partition $[,\ldots,l_i,\ldots,l_j,\ldots,l_{i+j},\ldots,]$ to the partition $[,\ldots,l_i-1,\ldots,l_j-1,\ldots,l_{i+j}+1,\ldots,]$ when a cluster of size i combines with a cluster of size j to form a cluster of size $i+j$. The factor $2il_ijl_j/N(N-1)$ is the probability of selecting an agent from a cluster of size i and another from a cluster of size j. The third term describes the increase in $P[l_1,\ldots,l_N]$ due to the change from the partition $[,\ldots,l_i+2,\ldots,l_{2i}-1,\ldots,]$ to $[,\ldots,l_i,\ldots,l_{2i},\ldots,]$. Similarly, the fourth term describes the change from the partition $[,\ldots,l_i+1,\ldots,l_j+1,\ldots,l_{i+j}-1,\ldots,]$ to $[,\ldots,l_i,\ldots,l_j,\ldots,l_{i+j},\ldots,]$. The last two terms describe the change in $P[l_1,\ldots,l_N]$ due to dissociations of clusters. The fifth term describes the change from the partition $[l_1,\ldots,l_i,\ldots,]$ to $[l_1+i,\ldots,l_i-1,\ldots,]$ when a cluster of size i dissolves. The factor il_i/N is the probability of selecting an agent from a cluster of size i. The last term describes the change from the partition $[l_1-i,\ldots,l_i+1,\ldots,]$ to $[l_1,\ldots,l_i,\ldots,]$. In this way, the dynamics in the EZ model is described as a flow of the probability function in a phase space consisting of all the possible partition of agents in a N-agent system.

Since $d/dt\Sigma_{[l_1,\ldots,l_N]}P[l_1,\ldots,l_N]=0$, a normalization condition can be introduced as

$$\sum_{[l_1,\ldots,l_N]} P[l_1,\ldots,l_N]=1. \tag{3}$$

In the stationary state,

$$\frac{d}{dt}P[l_1,\ldots,l_N]=0.$$

For small values of N, $P[l_1,\ldots,l_N]$ may be obtained by simply solving a set of algebraic equations. When N is large, solving $P[l_1,\ldots,l_N]$ directly becomes increasingly difficult. When N is large and a is not too small, Eq. (2) can be greatly simplified and allows an exact solution.

It is useful to define the quantities

$$\langle n_i\rangle=\sum_{[l_1,\ldots,l_N]} P[l_1,\ldots,l_N]l_i, \tag{4}$$

$$\langle n_in_j\rangle=\sum_{[l_1,\ldots,l_N]} P[l_1,\ldots,l_N]l_il_j, \tag{5}$$

and

$$\langle n_in_jn_k\rangle=\sum_{[l_1,\ldots,l_N]} P[l_1,\ldots,l_N]l_il_jl_k. \tag{6}$$

It follows from Eq. (1) that:

$$\sum_{i=1}^{N} i\langle n_i\rangle=N, \tag{7}$$

$$\sum_{i=1}^{N} i\langle n_i n_j\rangle = N\langle n_j\rangle, \tag{8}$$

$$\sum_{i=1}^{N} i\langle n_i n_j n_k\rangle = N\langle n_j n_k\rangle. \tag{9}$$

A mean-field theory can then be systematically developed. When N is sufficiently large and a is not too small, i.e., $a \gg 1/\sqrt{N}$, $\langle n_s\rangle$ is of the order of N for finite s. Since $\langle n_s\rangle$ decays rapidly with s, $\langle n_s\rangle$ for large values of s plays little role in the determination of $\langle n_p\rangle$ for small values of p. Therefore, if we are interested in extracting the s dependence of $\langle n_s\rangle$ for small s, we may establish a mean-field approximation by decoupling

$$\langle n_i n_j\rangle \approx \langle n_i\rangle\langle n_j\rangle, \tag{10}$$

which is valid when fluctuations are small. For $N\to\infty$ and $a \gg 1/\sqrt{N}$, the approximation of neglecting fluctuations can be justified self consistently [9].

Multiplying Eq. (2) by l_s and summing over all possible partitions $[l_1, \ldots, l_N]$, an equation for $\partial\langle n_s\rangle/\partial t$ is obtained. Using Eqs. (7)–(9) together with the mean-field approximation, one obtains for $s \geq 2$

$$\frac{\partial\langle n_s\rangle}{\partial t} = -\frac{(2-a)s\langle n_s\rangle}{N} + \frac{(1-a)}{N^2} \times \sum_{r=1}^{s-1} r\langle n_r\rangle (s-r)\langle N_{s-r}\rangle, \tag{11}$$

and for $s=1$

$$\frac{\partial\langle n_1\rangle}{\partial t} = -\frac{2(1-a)\langle n_1\rangle}{N} + \frac{a}{N}\sum_{r=2}^{N} r^2\langle n_r\rangle = -\frac{2(1-a)\langle n_1\rangle}{N} + \frac{a}{N}\sum_{r=2}^{\infty} r^2\langle n_r\rangle, \tag{12}$$

where we have used $1/(N-1)\approx 1/N$. These equations for $\langle n_s\rangle$ in the stationary state are identical to those analytically solved by D'Hulst and Rodgers [2]. Here, we re-covered these equations as an approximation to Eq. (2), which is the basic equation for the EZ model. The solution indicates that $\langle n_s\rangle$ is proportional to N and decays with a power law of exponent $\alpha = -5/2$, with an exponential cutoff showing up for large s. The result also justifies the assumption that $\langle n_s\rangle$ of small s are mainly determined by $\langle n_r\rangle$ of small r. Notice that an analytical extension has been made in the second equality in Eq. (12) which is valid only when $N \gg 1$ and $a \gg 1/\sqrt{N}$. In this case, $\langle n_r\rangle$ for $r\sim N$ is terminated by the exponential cutoff.

III. LARGE N AND $a \ll 1/(N \ln N)$ LIMIT: MEAN-FIELD APPROXIMATION

For large N and $a \ll 1/(N\ln N)$, the state of the system in which all agents combined to form a single cluster becomes dominantly important. The probability $P[0,0,\ldots,1]$ is almost equal to unity and the probabilities for the remaining partitions to occur are small and proportional to a. We define

$$A = \sum_{[l_1,\ldots,l_N]}{}' P[l_1,\ldots,l_N], \tag{13}$$

where Σ' denotes a summation over all possible partitions except $[0,0,\ldots,1]$. Apparently, $A\sim a$. On the other hand, $\langle n_s\rangle \sim a$ for $s<N$. We examine the size dependence of $\langle n_s\rangle$ for small s. Since the partition $[0,0,\ldots,1]$ can only be broken into the partition $[N,0,\ldots,0]$ in the dissociation of the largest possible cluster, one may expect that $\langle n_s\rangle/A \gg 1$ for small s. In other words, the average number l_s of clusters of size s for small s in important partitions $[l_1,\ldots,l_N]$ other than $[0,0,\ldots,1]$ is large. Therefore, the mean-field approximation may give a reasonable description of the behavior of $\langle n_s\rangle$ for small s. Since $A \ll 1$, the mean-field approximation now becomes

$$\langle n_i n_j\rangle \approx \langle n_i\rangle\langle n_j\rangle/A \tag{14}$$

for small i and j. Furthermore, one has

$$\sum_{s=1}^{N-1} s\langle n_s\rangle = AN \tag{15}$$

and

$$\sum_{s=1}^{N-1} s\langle n_i n_s\rangle = N\langle n_i\rangle. \tag{16}$$

Following similar procedures as in getting Eqs. (11) and (12), one obtains

$$\frac{\partial\langle n_s\rangle}{\partial t} = \frac{1}{N^2 A}\sum_{r=1}^{s-1} r(s-r)\langle n_r\rangle\langle n_{s-r}\rangle - \frac{2s\langle n_s\rangle}{N} \tag{17}$$

for $s \geq 2$, where terms proportional to a^2 are neglected. Equation (17) may be analytically extended to $s=\infty$ because $\langle n_s\rangle$ is very small for large s except $s=N$. The equation for $\langle n_1\rangle$ is slightly different. The dissociation of $[0,0,\ldots,1]$ is the dominant source of cluster of a single agent, and thus is crucial in the determination of $\langle n_1\rangle$. All other contributions to $\langle n_1\rangle$ are negligible in the limit of $a\to 0$. Thus,

$$\frac{\partial\langle n_1\rangle}{\partial t} = aN - \frac{2\langle n_1\rangle}{N}. \tag{18}$$

Stationary solution to Eqs. (17)–(18) can be found by the generating function approach [2]. Defining the generating function

$$g(\omega) = \sum_{s=2}^{\infty} s\langle n_s\rangle e^{-\omega s}, \tag{19}$$

it is straightforward to obtain from Eq. (17) that

$$g(\omega)=\frac{1}{2NA}[\langle n_1\rangle e^{-\omega}+g(\omega)]^2. \tag{20}$$

It also follows from Eq. (18) that in the stationary state

$$\langle n_1\rangle=\frac{N^2a}{2}. \tag{21}$$

Since $g(0)=NA-\langle n_1\rangle$ and $g(0)=NA/2$ from Eq. (20), we have

$$A=Na. \tag{22}$$

Substituting Eq. (22) into Eq. (20), one finds that

$$g(\omega)=\frac{N^2a}{2}(1-\sqrt{1-e^{-\omega}})^2, \tag{23}$$

and consequently

$$\langle n_s\rangle=N^2a\frac{(2s-2)!}{2^{2s-1}s!^2}. \tag{24}$$

For values of s that the Stirling's formula holds, the above equation gives

$$\langle n_s\rangle\approx N^2as^{-5/2}. \tag{25}$$

Comparing with the results in Ref. [2], we found that when $a\ll 1/(N\ln N)$, $\langle n_s\rangle$ still follows a power law with the exponent $-5/2$, but the coefficient in front is changed from N to $N^2a\ll N$. Therefore, the system almost becomes a single cluster consisting of all agents when $a\ll 1/(N\ln N)$. The probability in any other states is very small and is proportional to a. It may be interesting to note that the situation is analogous to that in Bose-Einstein condensation, in which the macroscopically occupied ground state corresponds to the situation of $\langle n_N\rangle=1-A\approx 1$ in the present problem.

IV. LARGE N AND $a\ll 1/(N\ln N)$ LIMIT: EXACT SOLUTION

When N is finite and a is small, one may obtain the solution for $\langle n_s\rangle$ for small s directly from Eq. (2). For instance,

$$\langle n_1\rangle=\frac{N^2a}{2}, \tag{26}$$

$$\langle n_2\rangle=\frac{(N-1)^2N^2a}{8(N-2)(2N-3)}, \tag{27}$$

and

$$\langle n_3\rangle=\frac{(N-1)^2N^2a}{18(N-3)(3N-7)}. \tag{28}$$

We found that for $\langle n_1\rangle$, the result is the same as the mean field result [Eq. (24)]. For $\langle n_2\rangle$ and $\langle n_3\rangle$, the mean-field results are different from Eqs. (27) and (28). The discrepancy suggests that for finite N, fluctuations may become significant. Interestingly as $N\to\infty$, Eqs. (26)–(28) reduce to $\langle n_1\rangle\to N^2a/2$, $\langle n_2\rangle\to N^2a/16$, and $\langle n_3\rangle\to N^2a/54$. These results are the same as the mean-field results for $\langle n_1\rangle$ and $\langle n_2\rangle$, but not for $\langle n_3\rangle$. Therefore, the effects of fluctuations are important even when $N\to\infty$ in the case of $a\ll 1/(N\ln N)$. In this case, the solutions to a set of hierarchical quantities $\langle n_i\rangle$, $\langle n_in_j\rangle$, $\langle n_in_jn_k\rangle$, etc., are needed in order to obtain an exact solution. To do so, we define

$$\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle=\sum_{l_1,\ldots,l_N}P[l_1,\ldots,l_N]l_1^{m_1}\cdots l_i^{m_i},\ldots, \tag{29}$$

and we have

$$\sum_{i=1}^{N}i\langle n_1^{m_1}\cdots n_i^{m_i+1}\cdots\rangle=N\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle. \tag{30}$$

We consider the quantities $\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle$ with

$$M_N=\sum_{i=1}^{N}im_i\ll N.$$

If we define

$$\overline{\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle}=\lim_{N\to\infty}\lim_{a\to 0}\frac{\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle}{N^{M+1}a},$$

then for $N\to\infty$ and $a\ll 1/(N\ln N)$, it can be shown that

$$\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle=N^{M+1}a\overline{\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle}+O(N^Ma), \tag{31}$$

where $M=\sum_{i=1}^{N}m_i$ and $\overline{\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle}$ is independent of N and a. Multiplying Eq. (2) by $l_1^{m_1}\cdots l_i^{m_i}\cdots$ and summing over all possible partitions $[l_1,\ldots,l_N]$, one obtains

$$\begin{aligned}2M_N\overline{\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle}&=\sum_{s=2}^{N}m_s\sum_{r=1}^{s-1}r(s-r)\\&\quad\times\overline{\langle n_1^{m_1}\cdots n_r^{m_r+1}\cdots n_{s-r}^{m_{s-r}+1}\cdots n_s^{m_s-1}\cdots\rangle}\\&\quad+\delta(m_2)\cdots\delta(m_N),\end{aligned} \tag{32}$$

where we have neglected the higher order terms in Eq. (31) and used $M_N\ll N$ and $1/(N-1)\approx 1/N$. Eq. (32) can be solved readily when M_N is not large to get

$$\overline{\langle n_1^{m_1}\cdots n_i^{m_i}\cdots\rangle}=\frac{(M_N-M)!}{(2M_N)^{M_N-M+1}}\prod_{i=1}^{N}\left(\frac{C_i}{(i-1)!}\right)^{m_i}, \tag{33}$$

where $C_i=4(2i)^{i-3}$. Hence, for small s we have

$$\langle n_s\rangle=\frac{N^2a}{2s^3}+O(Na). \tag{34}$$

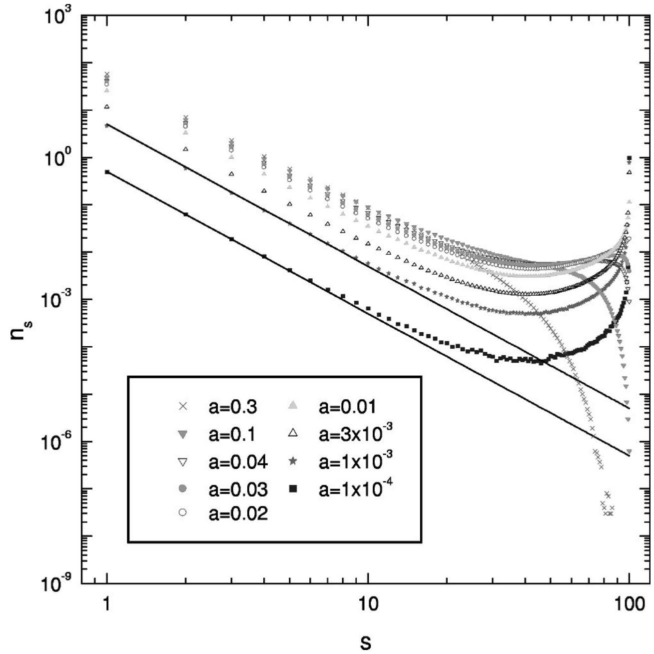

FIG. 1. The number of clusters $n(s)$ with size s as a function of the cluster size s for different values of a in a system with $N=100$. The data are obtained by averaging over ten runs with each run corresponding to a different initial configuration lasting for 10^7 time steps. The solid lines give the analytical result of $n_s=N^2a/2s^3$ [Eq. (34)], for $a=10^{-3}$ and $a=10^{-4}$.

Thus, for small s, $\langle n_s\rangle$ obeys a power law with an exponent -3, a result different from the mean-field treatment. The coefficient in front of the s^{-3} behavior is still given by the coefficient N^2a.

Term of order $O(Na)$ neglected in Eq. (34) is unimportant for small s, but it may become significant when s becomes larger. The following consideration serves to illustrate the point. Neglecting the term $O(Na)$, one obtains

$$A=\frac{1}{N}\sum_{s=1}^{N-1} s\langle n_s\rangle=\frac{\pi^2}{12}Na<Na.$$

On the other hand, Eq. (33), in the limit $N\to\infty$ and $a \ll 1/(N\ln N)$, gives

$$\langle n_1^s\rangle=\frac{N^{s+1}a}{2s}.$$

From the Schwartz inequality $A\langle n_1^2\rangle>\langle n_1\rangle^2$, one has

$$A>Na.$$

Therefore, the neglected terms in Eq. (34), although not important for small s, become significant for large s and give an additional contribution to A. We would also like to point out that results of numerical simulation suggest that $A\sim CNa\ln N$ with $C\sim 0.5$. The mean-field result of $A=Na$ is therefore inconsistent with numerical results. Therefore, numerical results suggest that the neglected term in Eq. (34) becomes important when s is large.

V. NUMERICAL SIMULATIONS

We carried out numerical simulations on the model for $N=100$ with different values of a corresponding to different regimes. A total number of 10^7 time steps were used in each run. Figure 1 shows the results of $\langle n_s\rangle$ as a function of s for $a=0.3$, 0.1, 0.04, 0.03, 0.02, 0.01, 3×10^{-3}, 10^{-3}, and 10^{-4}, respectively. The data are obtained by averaging over ten runs, with each run corresponding to a different initial configuration.

When $a=10^{-3}$ and 10^{-4}, $\langle n_s\rangle$ decays as a power law with the exponent -3 for small s. However, the s dependence of $\langle n_s\rangle$ deviates from a power law for large s. In particular, when $s>40$, $\langle n_s\rangle$ increases with s . This result indicates that the neglected term in Eq. (34) is important when s is large. Notice that there is a jump between $\langle n_{N-1}\rangle$ and $\langle n_N\rangle$ for $a=10^{-4}$. This can be understood by recalling that as $a\to 0$, $\langle n_N\rangle\to 1$ but $\langle n_{N-1}\rangle\sim a$. The jump is not apparent for $a=10^{-3}$ because the value is not sufficiently small. Hence, the results for small a ($a=10^{-3},10^{-4}$) in Fig. 1 are consistent with the exact results presented in Sec. IV, but inconsistent with the mean-field result in Sec. III. The behavior of the neglected term in Eq. (34) for large s is worth further investigation.

Some quantitative features of the numerical results are also worth pointing out. Recall that $A=1-P[0,0,\ldots,1]=1-\langle n_N\rangle$. For $N=100$, we get $\langle n_N\rangle=0.78$ for $a=10^{-3}$ and $\langle n_N\rangle=0.975$ for $a=10^{-4}$. Therefore, $A\sim 2Na$ for $N=100$. This result is inconsistent with the mean-field theory, which predicts $A=Na$. Since $A\sim a$ as $a\to 0$, we also carried out numerical simulations for $N=10$, 31, 100, 310, and 1000 so as to investigate the N dependence of A. Extrapolation of our numerical results indicates that $A\approx 0.5N\ln Na$. Therefore, only when $a\ll 1/(N\ln N)$, we have $A\ll 1$ and $P[0,0,\ldots,1]\approx 1$ so that the discussions in Secs. III and IV are valid. Also, the numerical results $\langle n_1\rangle=0.494$, $\langle n_2\rangle=0.062$, $\langle n_3\rangle=0.019$ for $N=100$ and $a=10^{-4}$ are in good agreement with the exact results $\langle n_1\rangle=N^2a/2=1/2$, $\langle n_2\rangle=1/16$, and $\langle n_3\rangle=1/54$. For $a=10^{-3}$, the numerical results of $\langle n_1\rangle=4.594$ is slightly off the exact result of $\langle n_1\rangle=N^2a/2=5$. This indicates that $a=10^{-3}$ is not sufficiently small for the case of $N=100$, and it should better be treated as a case in the crossover regime.

For comparison, Fig. 1 also includes results for values of a in the intermediate regime of $1/(N\ln N)<a<1/\sqrt{N}$. It can be seen that the s dependence of $\langle n_s\rangle$ in the intermediate regime ($a=0.04$, 0.03, 0.02, and 0.01) is rather complicated. The transition from a "subcritical" behavior characterized by a power law with an exponential cutoff to a "supercritical" behavior characterized by a power law with a bump seems to be gradual. If one is to locate a critical value a_c at which the competition between the two regimes balances, a_c would lie in the range $0.02<a_c<0.03$ for the system studied. For $a=0.02$, the numerical results basically follow the "supercritical" behavior. For $a=0.03$, $\langle n_s\rangle$ follows a power law for small s and starts to show "supercritical" behavior for $40<s<80$ but eventually goes over to the "subcritical" behavior for $s>80$. Note that the results for $a=0.3$ are typical for cases with $a>1/\sqrt{N}$.

VI. DISCUSSIONS

We have shown that finite-size effect is significant for $a \ll 1/(N \ln N)$ in the EZ model. In this case, the system almost becomes a single cluster containing all the agents. On the other hand, finite-size effect is not important when N is sufficiently large and $a \gg 1/\sqrt{N}$. In this case, all $\langle n_s \rangle$ with s being the order of N are negligibly small. Therefore, $\langle n_s \rangle$ for finite and small s is mainly determined by $\langle n_r \rangle$ for finite r.

The finite-size effect plays an important role in the determination of $\langle n_s \rangle$ for finite s in the limit of $a \ll 1/(N \ln N)$. Although $\langle n_s \rangle$ decays as a power law with s both in the cases of $a \ll 1/(N \ln N)$ and $a \gg 1/\sqrt{N}$ (in addition to an exponential cutoff) within a mean field approach, the coefficients in front of the $s^{-5/2}$ behavior are different. For $a \gg 1/\sqrt{N}$, the coefficient is N. For $a \ll 1/(N \ln N)$, the coefficient becomes $N^2 a$, which is much less than N.

We also studied the validity of the mean-field approximation. It is possible, in a self-consistent way, to show that [9] the mean-field approximation is valid in the case of $N \to \infty$ and $a \gg 1/\sqrt{N}$. However, fluctuation effects are important for $a \ll 1/(N \ln N)$. The direct consequence of the fluctuation effects is to change the exponent of $\langle n_s \rangle$ from $-5/2$ in the mean-field approximation to -3 for small s.

It is interesting to discuss the intermediate regime in which $1/(N \ln N) < a < 1/\sqrt{N}$. When $a \gg 1/\sqrt{N}$, $\langle n_s \rangle$ with $s \sim N$ are negligibly small because of the exponential cutoff. The analytic theory developed by D'Hulst and Rodgers [2] is valid. When $a < 1/\sqrt{N}$, the exponential cutoff in $\langle n_s \rangle$ does not terminate $\langle n_s \rangle$ when $s \sim N$. In other words, those $\langle n_r \rangle$ with $r \sim N$ also play an important role in the determination of $\langle n_s \rangle$ for small s. Mathematically speaking, the second term on the right-hand side of Eq. (12) should be replaced by $(a/N)\Sigma_{r=2}^{N} r^2 \langle n_r \rangle$. Consequently, $\langle n_1 \rangle$ is less than the result $N/2$ obtained in Ref. [2]. As a decreases further, the finite-size effect becomes significant. When $a \sim 1/(N \ln N)$, the probability $P[0,0, \ldots ,1] = 1 - A$ becomes finite and the probabilities in other partitions are small and proportional to a. Finally when $a \ll 1/(N \ln N)$, $P[0,0, \ldots ,1] = 1 - A \approx 1$ and the theory presented in Sec. IV is valid. In the intermediate regime of $1/(N \ln N) < a < 1/\sqrt{N}$, numerical results showed that the s dependence of $\langle n_s \rangle$ is rather complicated. Further work is needed to study the detail behavior of the transition between a "subcritical" behavior characterized by a power law with an exponential cutoff to a "supercritical" behavior characterized by a power law with a bump.

Finally, the importance of the neglected term in the exact result [Eq. (34)] was studied. When s is small and N is large, the neglected term is negligible and $\langle n_s \rangle$ is exactly described by a power law with the exponent -3 when $a \to 0$. However, numerical simulations indicate that while $\langle n_s \rangle \sim s^{-3}$ for small s, $\langle n_s \rangle$ increases with s when $s \sim N$. This result implies that the neglected term becomes dominantly important when $s \sim N$. The behavior in this regime deserves further investigations.

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[1] V.M. Eguíluz and M.G. Zimmermann, Phys. Rev. Lett. **85**, 5659 (2000).
[2] R. D'Hulst and G.J. Rodgers, Eur. Phys. J. B **20**, 619 (2001).
[3] D. Zheng, P.M. Hui, and N.F. Johnson, cond-mat/0105474.
[4] D. Zheng, G.J. Rodgers, P.M. Hui, and R. D'Hulst, Physica A **303**, 176 (2002).
[5] See, for example, R.N. Mantegna and H.E. Stanley, *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, Cambridge, England, 2000).
[6] J.D. Farmer, Comput. Sci. Eng. **1**, 26 (1999).
[7] R. Cont and J.-P. Bouchard, Macroecon. Dyn. **4**, 170 (2000).
[8] See, for example, L.E. Reichl, *A Modern Course in Statistical Physics* (University of Texas Press, 1980), Chap. 6.
[9] Y.B. Xie and B.H. Wang (unpublished).